2026

MATHEMATICS

Full Marks : 100

Pass Marks : 33

Time : Three hours

Attempt all Questions.

The figures in the right margin indicate full marks for the questions.

For Question Nos. 1–10, write the letter associated with the correct answer.

1. Let $f: R \to R$ be defined by 1

$$f(x) = \begin{cases} 1 & \text{if } x \in Q \\ -1 & \text{if } x \notin Q \end{cases}, \text{ then } f\left(\frac{1}{2}\right) + f(\pi) + f\left(\sqrt{2}\right) =$$

(A) 0 (B) 1

(C) –1 (D) 2

2. $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ is equal to 1

(A) 1 (B) $\frac{1}{3}$

(C) –1 (D) $\frac{1}{2}$

3. If $A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$, then A^{2026} is equal to 1

(A) $\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$ (B) $\begin{bmatrix}0 & 2026\\ 0 & 0\end{bmatrix}$

(C) $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ (D) $\begin{bmatrix}2026 & 0\\ 0 & 2026\end{bmatrix}$

4. If $y=\log\left(\sin e^{x}\right)$, then $\frac{dy}{dx}$ is equal to 1

(A) $\cot e^{x}$ (B) $\operatorname{cosec} e^{x}$

(C) $e^{x}\cot e^{x}$ (D) $e^{x}\operatorname{cosec} e^{x}$

5. $\int e^{5\log x}dx$ is equal to 1

(A) $\frac{x^5}{5}+C$ (B) $\frac{x^6}{6}+C$

(C) $5x^4+C$ (D) $6x^5+C$

6. The solution of the differential equation $(x^2+1)\frac{dy}{dx}=1,\ y(1)=\frac{\pi}{2}$ is 1

(A) $y=\tan^{-1}x+\frac{\pi}{3}$ (B) $y=\tan^{-1}x$

(C) $y=\tan^{-1}x+\frac{\pi}{6}$ (D) $y=\tan^{-1}x+\frac{\pi}{4}$

7. The scalar projection of the vector $3\hat{i}-\hat{j}-2\hat{k}$ on the vector $\hat{i}+2\hat{j}-3\hat{k}$ is 1

(A) $\frac{7}{\sqrt{14}}$ (B) $\frac{7}{14}$

(C) $\frac{6}{13}$ (D) $\frac{7}{2}$

8. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}|=2, |\vec{b}|=7$ and $\vec{a}\times\vec{b}=3\hat{i}+2\hat{j}+6\hat{k}$

then the angle between $\vec{a}$ and $\vec{b}$ is 1

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

9. The two lines $x = ay + b, z = cy + d$

and $x = a_1y + b_1, z = c_1y + d_1$

are perpendicular to each other, if 1

(A) $\frac{a}{a_1}+\frac{c}{c_1}=1$ (B) $\frac{a}{a_1}+\frac{c}{c_1}=-1$

(C) $aa_1 + cc_1 = 1$ (D) $aa_1 + cc_1 = -1$

10. The objective function $z = ax + by$ of a LPP has maximum value 42 at (4,6) and minimum value 19 at (3,2). Which of the following is true? 1

(A) $a = 9, b = 1$ (B) $a = 9, b = 2$

(C) $a = 3, b = 5$ (D) $a = 5, b = 3$

Question numbers 11 and 12 are assertion and reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Answer these two questions by selecting the appropriate option given below :

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true and R is not the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

11. Assertion (A) : $f(x)=x^4$ is decreasing in the interval $(0,\infty)$ 1

Reason (R) : Any derivable function $y=f(x)$ is decreasing if $\frac{dy}{dx}<0$

12. Assertion (A) : If a line makes angles α,β,γ with positive direction of the coordinate axes then $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$ 1

Reason (R) : The sum of the squares of the direction cosines of a line is one.

13. Evaluate : $\sin\left\{\cos^{-1}\left(-\frac{4}{5}\right)\right\}$ 1

14. If A = [1 2 5 7] and B = $\begin{bmatrix}6\\2\\4\\8\end{bmatrix}$ write the orders of AB and BA 1

15. Show that the matrix B'AB is symmetric if A is symmetric. 1

16. Differentiate e^x with respect to $\log x$. 1

17. The total revenue in Rupees received from the sale of x units of a product is given by $R(x)=13x^2+26x+15$

Find the marginal revenue when $x = 7$ 1

18. If $\int_0^a \frac{dx}{1+4x^2}=\frac{\pi}{8}$, then find the value of a. 1

19. How many arbitary constants are there in the particular solution of the differential equation. $\frac{dy}{dx}=-4xy^2$; $y(0) = 1$? 1

20. What is meant by bounded region of a LLP? 1

21. Show that the function $f: R \to R$ given by $f(x) = x^3$ is injective. 2

22. Check if the relation R in the set **R** of real numbers defined as

$R = \{(a,b): a < b\}$ is (i) Symmetric and (ii) Transitive 2

23. If $y = 500e^{7x} + 600e^{-7x}$, show that $\frac{d^2y}{dx^2} = 49y$ 2

24. (a) Find the area of a Parallelogram whose adjacent sides are determined by the vectors 2

$$\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$$
$$\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$$

OR

(b) If $\hat{a}$ and $\hat{b}$ are unit vectors, then prove that $|\hat{a} + \hat{b}| = 2\cos\frac{\theta}{2}$ where θ is the angle between them.

25. If A and B are two events such that $P(A) = 0.4, P(B) = 0.8 \text{ and } P(B/A) = 0.6$, then find *P(A/B)*. 2

26. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, "the number is even" and B be the event, "the number is red". Are A and B independent? 2

27. (a) If $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x$, then find the value of x 3

OR

(b) Prove that : $\sin^{-1}\left(\frac{8}{17}\right) + \cos^{-1}\left(\frac{4}{5}\right) = \cot^{-1}\left(\frac{36}{77}\right)$

28. (a) Solve the differential equation : $ydx+\left(x-y^2\right)dy=0$ 3

OR

(b) Find the general solution of the differential equation

$$\left(xy-x^2\right)dy=y^2dx$$

29. The volume of the spherical balloon being inflated changes at a constant rate. If initially its radius 3 units and after 3 seconds it is 6 units. Find the radius of the balloon after 3 seconds. 3

30. Find the angle between the lines 3

$$\vec{r}=2\hat{i}-5\hat{j}+\hat{k}+\lambda\left(3\hat{i}+2\hat{j}+6\hat{k}\right)$$

and $\vec{r}=7\hat{i}-6\hat{j}-6\hat{k}+\mu\left(\hat{i}+2\hat{j}+2\hat{k}\right)$

31. If the inverse of a square matrix exists, prove that it is unique. If A and B are both invertible square matrices of the same order, prove that

$(AB)^{-1} = B^{-1} A^{-1}$ 4

32. (a) Sketch the region $\left\{(x,y); y=\sqrt{4-x^2}\right\}$ and x-axis. Find the area of the region using integration. 4

OR

(b) Using integration, find the area bounded by the line $y = x$, the x-axis and lines $x = -1$ & $x = 2$.

33. Find the vector and cartesian equations of a line passing through the point A (1, 2, –1) and parallel to the lines $5x - 25 = 14 - 7y = 35z$. 4

34. Solve the following LPP graphically : 4

Minimise $Z = 13x - 15y$, subject to the constraints

$x + y \leq 7$

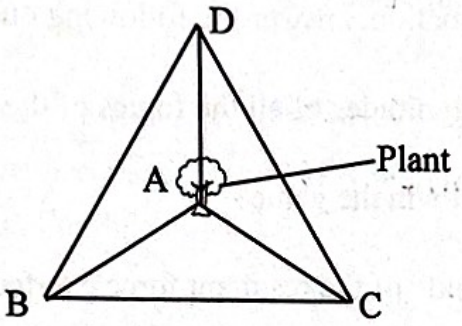
$2x - 3y + 6 \geq 0$

$x \geq 0, y \geq 0$

(Graph paper will not be supplied)

Case study - based question

35. (a) Thoiba purchased an air plant holder which is in shape of tetrahedron. Let A, B, C, D be the vertices of the air plant holder where A (1, 2, 3), B (3, 2, 1), C (2, 1, 2), D (3, 4, 3)



Base on the above information, answer the following questions :

(i) Find the vector $\overrightarrow{BC}$

(ii) Find the vector $\overrightarrow{BD}$

(iii) Find the area (ΔBCD) 1+1+2=4

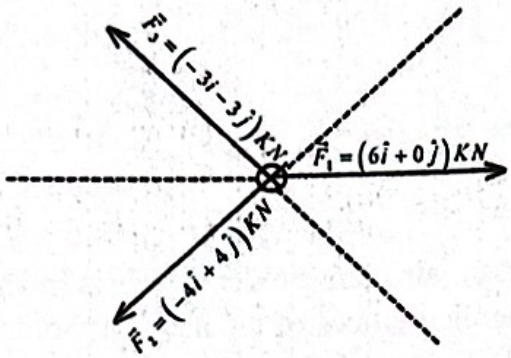
OR

(b) Teams A, B, C went to play a tug of war game. Team A, B, C have attached a rope to a metal ring and is trying to pull the ring into own area.

Team A pulls with force $\vec{F}_1 = \left(6\hat{i} + 0\hat{j}\right) KN$

Team B pull with force $\vec{F}_2 = \left(-4\hat{i} + 4\hat{j}\right) KN$

Team C pull with force $\vec{F}_3 = \left(-3i - 3\hat{j}\right) KN$



Read the above intesmation, answer the following questions.

(i) What are the magnitudes of all the forces of the three teams?

(ii) Which team will win the game?

(iii) Find the magnitude of the resultant force exerted by the teams.

36. Given that $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ *and* $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ 6

Verify that AB = 8I and hence solve the system of equations

$$x - y + z = 4$$
$$x - 2y - 2z = 9$$
$$2x + y + 3z = 1$$

37. If $f(x)=\begin{cases}\dfrac{\sin(a+1)x+2\sin x}{x} & ,x<0\\ 2 & ,x=0\\ \dfrac{\sqrt{1+bx}-1}{x} & ,x>0\end{cases}$

is continuous at $x=0$, find the values of a and b. 6

38. (a) Given the sum of the perimeters of a square and a circle, show that the sum of their areas is least when the side of the square is equal to twice the radius of the circle. 6

OR

(b) The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm/sec. How fast is the area decreasing, when the two sides are equal to the base ?

39. (a) Evaluate $\int_0^{\frac{\pi}{4}}\frac{\sin x+\cos x}{16+9\sin 2x}dx$ 6

OR

(b) Prove that $\int_0^{\frac{\pi}{2}}(2\log\sin x-\log\sin 2x)\,dx=\frac{\pi}{2}\log\frac{1}{2}$

40. Prove that $\int_0^{\frac{\pi}{2}}\frac{\sin^2 x}{\sin x+\cos x}\,dx=\frac{1}{\sqrt{2}}\log\left(\sqrt{2}+1\right)$ 6

41. (a) In a factory which manufactures bolts, machines A, B and C manufacture respectively, 30%, 50% and 20% of the bolts. Of their outputs 3%, 4% and 1% are respectively difective bolts. A bolt is drawn at random from the product and is found to be defective. Find the probability that this is not manufactured by machine C. 6

OR

(b) A shopkeeper sells three types of flower seeds A_1, A_2 and A_3. They are sold with mixture where the proportions are 4 : 4 : 2 respectively. The germination rates of three types of seeds are 45%, 60% and 35% respectively.

(i) Find the probability of a randomly choosen seed to germinate.

(ii) If the randomly choosen seed does not germinate, find the probability that it is of type A_2.