Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME - SEMESTER– 1 EXAMINATION – SUMMER - 2023

Subject Code:3712017 **Date:15 Jul 2023**

Subject Name: Advanced Solid Mechanics

Time:02:30 PM TO 05:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Derive strain displacement relation in plane polar coordinate system with neat sketch. **07**

(b) Derive any one basic differential equation of equilibrium Cartesian coordinate with neat sketch and write the other two equations. **07**

Q.2 **(a)** Derive any one basic differential equations of equilibrium in 2-D Polar Coordinate System with neat sketch. **07**

(b) Differentiate plane stress and plain strain problem with suitable examples. **07**

OR

(b) Explain assumptions made in Solid Mechanics and explain "Generalized Hooke's Law". **07**

Q.3 **(a)** Derive strain displacement relation in Cartesian co-ordinate system with neat sketch. **07**

(b) For the following state of stresses, find the principal stresses and the direction cosines of any ONE principal stress. **07**

Normal stresses :σ_{xx} = 400 MPa, σ_{yy} = 100 MPa, σ_{zz} = -50 MPa

Shear Stresses : τ_{xy} = 60 MPa, τ_{yz} = -20 MPa, τ_{zx} = -50 MPa

OR

Q.3 **(a)** Derive the following equation with usual notations. **07**

$$\varepsilon_\theta = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta + \varepsilon_{xy}\sin 2\theta$$

(b) Using Swift construction, find normal and resultant shear stress on a plane whose normal has directions cosines are l = 0.342, m = 0.544 and n = 0.766 respectively w.r.t. Principal stresses: P_1 = 700 MPa (Tensile), P_2 = 250 MPa (Tensile) and P_3 = 300 MPa (Comp). **07**

Q.4 **(a)** Give characteristics of Airy's stress function. **07**

(b) Show that the following 2-D state of stresses without body forces is in equilibrium: **07**

$$\sigma_X = 3x^2 + 4xy - 8y^2 \quad \sigma_Y = 2x^2 + xy + 3y \quad \tau_{XY} = -\tfrac{1}{2}x^2 - 6xy - 2y^2$$

OR

Q.4 **(a)** Write the compatibility equation for 2-D strains. From which derive the equation $\nabla^4\phi = 0$, where ϕ represents Airy's stress function. **07**

(b) Find the strains induced in 'r' and 'θ' direction if the displacements '*u*' and '*v*' along the same are given by **07**

$$u = 7\,r\cos\theta + \frac{5}{2}\,r\sin\theta$$

$$v = 4\,r - \frac{5}{2}\,r\cos\theta + 7\,r\sin\theta$$

Q.5 **(a)** Enlist the assumptions involved in the theory of torsion of a long bar. Which coordinate system will you prefer to solve problem related to torsion? Justify your answer. **07**

(b) Elaborate soap film bubble analogy and derive the following equation with usual notations: **07**

$\nabla^2 (\varphi/2G\theta) = \nabla^2 (sz/p) = -1.$

OR

Q.5 **(a)** State and explain Saint Vanant's principle. **07**

(b) A cylinder 120 mm internal diameter is subjected to an internal pressure 50 MPa. There is no external pressure. If the allowable stress in the metal is 150 MPa. Determine external diameter. **07**